Tribhuvan University
Institute of Engineering
Pulchowk Campus
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Unit: III- Control Strategies

Class-09:
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Presented by
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Subject Name
EE: Modelling and Control of Electric Drives

Discussed in the Previous Class

In the previous class discussed the following topics:

- ❖ Overview of Types of DC Motor Drives
- ❖ Speed Control of DC Motor Drives
- ❖ Armature Voltage Control of DC Motor using Transformer
- ❖ Controlled Rectifier Fed DC Drives

Lecture Outcomes

- ❖ Single Phase Fully Controlled Rectifier Control of DC Motor
- ❖ Single Phase Half Controlled Rectifier Control of DC Motor
- ❖ Lecture remarks: Key points of today's class

Single Phase Fully Controlled Rectifier Control of DC Motor

- The Single Phase Fully Controlled Rectifier Control of DC Motor is shown in Fig. 1.

- Motor is shown by its equivalent circuit. Field supply is not shown. When field control is required, field is fed from a controlled rectifier, otherwise from an uncontrolled rectifier.

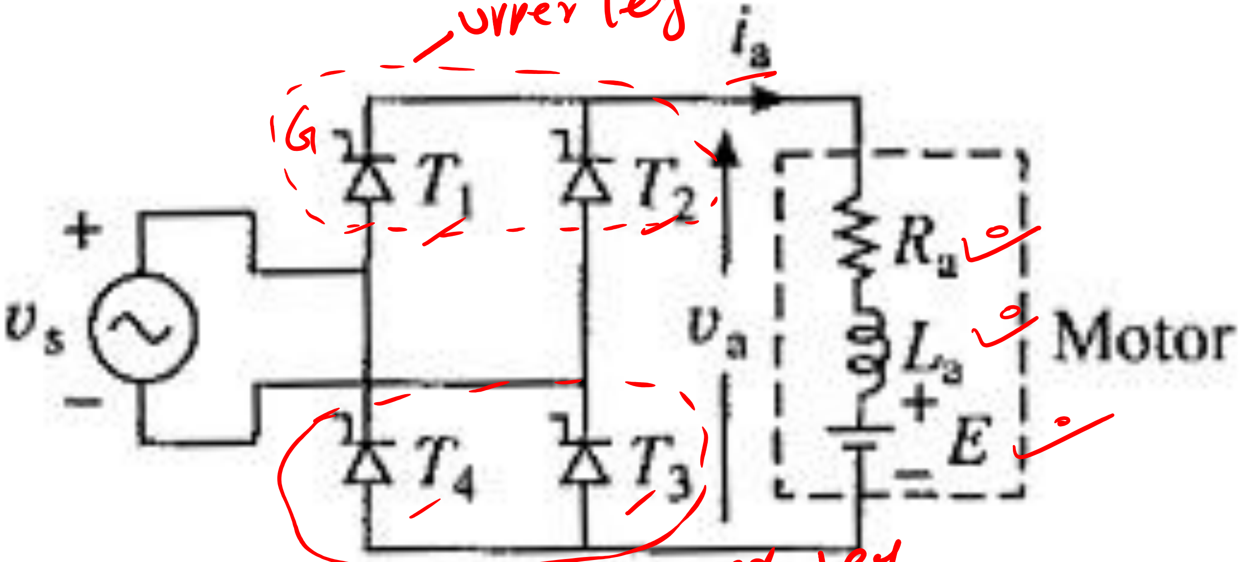


Fig. 1. Drive Circuit.

Single Phase Fully Controlled Rectifier Control of DC Motor

- The ac input voltage is defined by

$$v_s = V_m \sin \omega t \qquad (1)$$

- In a cycle of source voltage, thyristors T_1 and T_3 are given gate signals from α to π, and thyristors T_2 and T_4 are given gate signals from (π + α) to 2π.

- *When the armature current does not flow continuously, the motor is said to operate in discontinuous conduction.*

- *When current flows continuously, the conduction is said to be continuous.*

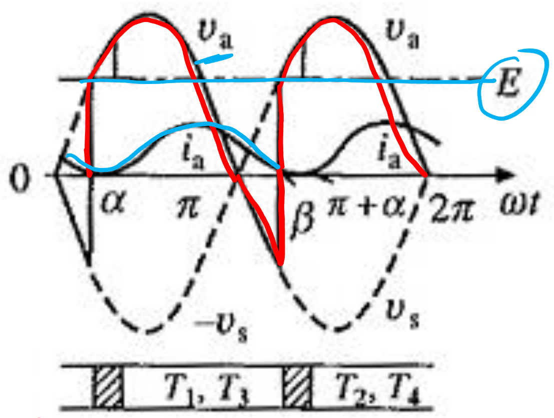


Fig. 2. Discontinuous conduction waveform.

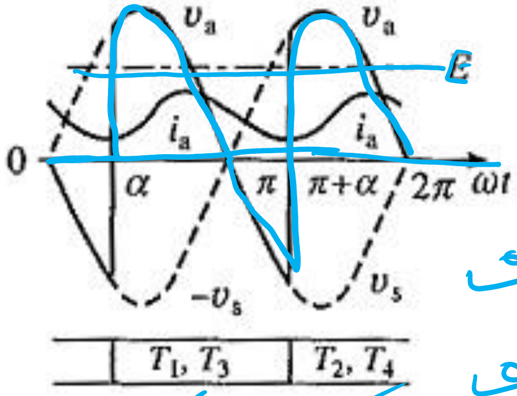


Fig. 3. Continuous conduction waveform.

Single Phase Fully Controlled Rectifier Control of DC Motor

- The drive under consideration, predominantly operates in discontinuous conduction.
- Discontinuous conduction has several modes of operation. The approximate, but simple, method of analysis is obtained when only the dominant mode of discontinuous conduction is taken into account.
- The motor terminal voltage and current waveforms for the dominant discontinuous conduction and continuous conduction modes are shown in Figs. 2 and 3.

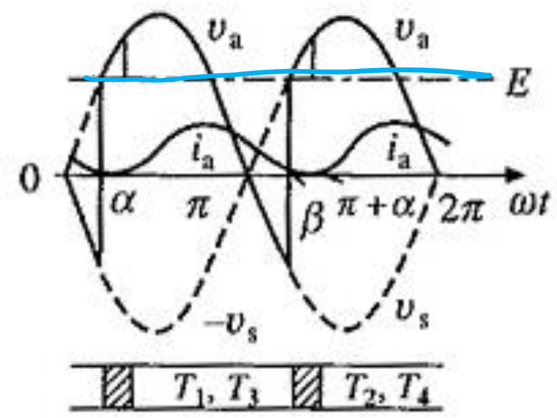


Fig. 2. Discontinuous conduction waveform.

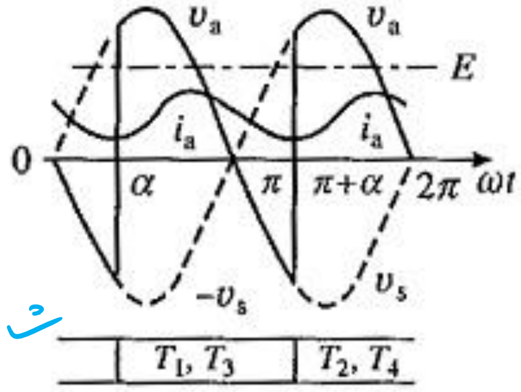


Fig. 3. Continuous conduction waveform.

Single Phase Fully Controlled Rectifier Control of DC Motor

- In discontinuous conduction mode of Single Phase Fully Controlled Rectifier Control of DC Motor, current starts flowing with the turn-on of thyristors T_1 and T_3 at $\omega t = \alpha$.
- Motor gets connected to the source and its terminal voltage equals v_s. The current, which flows against both, E and the source voltage after $\omega t = \pi$, falls to zero at β.
- Due to the absence of current T_1 and T_3 turn-off. Motor terminal voltage is now equal to its induced voltage E. When thyristors T_2 and T_4 are fired at $(\pi + \alpha)$, next cycle of the motor terminal voltage v_a starts.

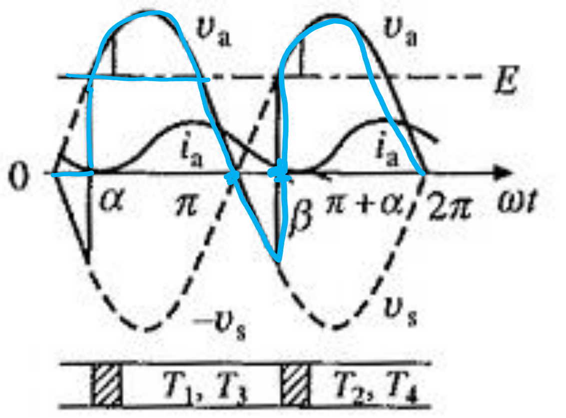


Fig. 2. Discontinuous conduction waveform.

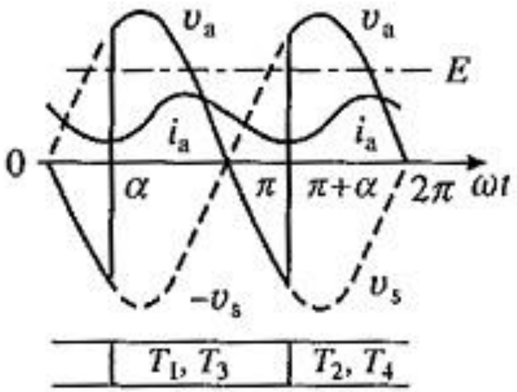


Fig. 3. Continuous conduction waveform.

Single Phase Fully Controlled Rectifier Control of DC Motor

- In continuous conduction mode of Single Phase Fully Controlled Rectifier Control of DC Motor, a positive current flows through the motor, and T_2 and T_4 are in conduction just before α.

- Application of gate pulses turns on forward biased thyristors T_1 and T_3 at α. Conduction of T_1 and T_3 reverse biases T_2 and T_4 and turns them off.

- A cycle of v_a is completed when T_2 and T_4 are turned-on at (π + α) causing turn-off of T_1 and T_3.

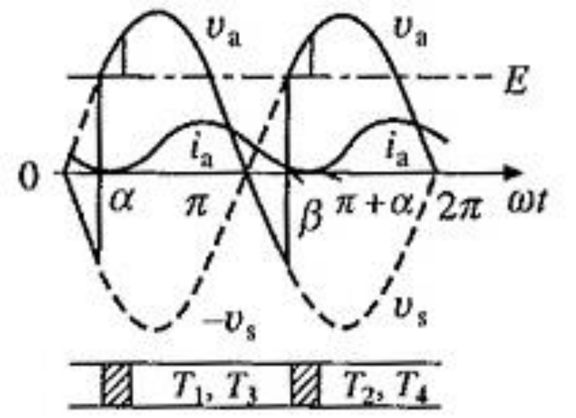


Fig. 2. Discontinuous conduction waveform.

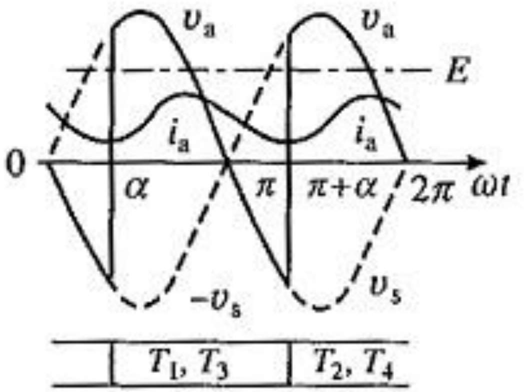


Fig. 3. Continuous conduction waveform.

Single Phase Fully Controlled Rectifier Control of DC Motor

Since armature current i_s is not perfect dc, the motor torque fluctuates. Since torque fluctuates at a frequency of 100 Hz, motor inertia is able to filter out the fluctuations, giving nearly a constant speed and ripple less E.

DC ripples

$T \propto i$

$V \propto N$

Reduced ripples

controlling firing pulses

⇓

Design controlled Algorithms

⇓

Power Electronics switches.

Discontinuous Conduction Mode

Single Phase Fully Controlled Rectifier Control of DC Motor

- In a Single Phase Fully Controlled Rectifier Control of DC Motor terminal voltage v_a, the drive operates in two intervals (Fig. 2):

1. Duty interval ($\alpha \leq \omega t \leq \beta$) when motor is connected to the source and $v_a = v_s$.

2. Zero current interval ($\beta \leq \omega t \leq \pi + \alpha$) when $i_a = 0$ and $v_a = E$.

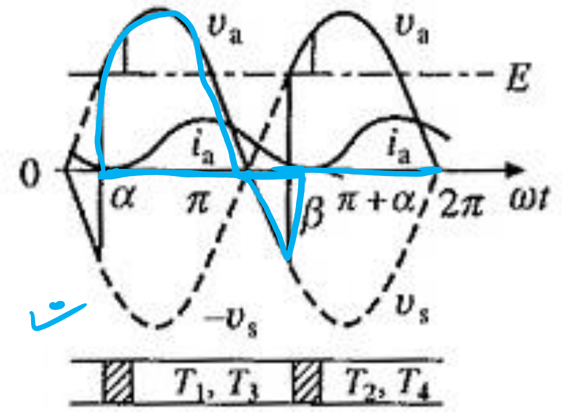
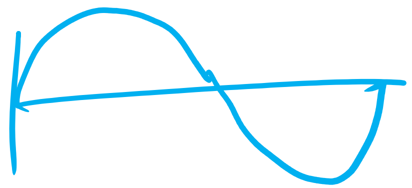


Fig. 2. Discontinuous conduction waveform.

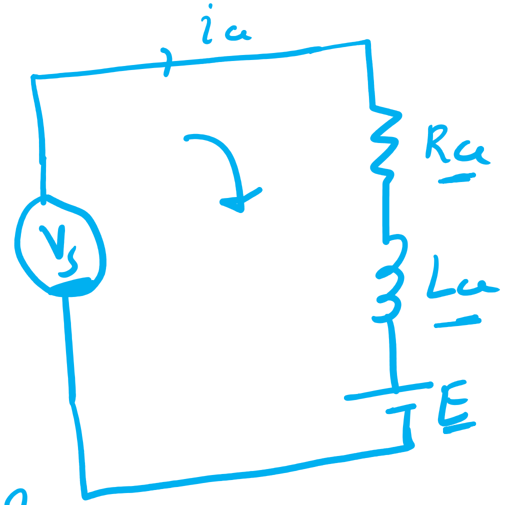


Single Phase Fully Controlled Rectifier Control of DC Motor

Drive operation is described by the following equations:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + E = V_m \sin \omega t, \text{ for } \alpha \le \omega t \le \beta \qquad (2)$$

$$v_a = E \quad \text{and} \quad i_a = 0 \quad \text{for} \quad \beta \le \omega t \le \pi + \alpha \qquad (3)$$



The solution of Eq. (2) has two components:

1. one due to the ac source $(V_m/Z) \sin(\omega t - \Phi)$, and
2. other due to back emf $(-E/R_a)$.

Each of these components has in turn a transient component. Let these be represented by a single exponent $K_1 e^{-t/\tau a}$, then

Single Phase Fully Controlled Rectifier Control of DC Motor

$$i_a(\omega t) = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R_a} + K_1 e^{-t/\tau_a} \quad \text{for } \alpha \le \omega t \le \beta \tag{4}$$

where

$$Z = \sqrt{R_a^2 + (\omega L_a)^2} \tag{5}$$

$$\phi = \tan^{-1}(\omega L_a / R_a) \tag{6}$$

Constant K_1 can be evaluated subjecting Eq. (4) to the initial condition $i_a(\alpha) = 0$.

Substituting the value of K_1 so obtained in Eq. (4) yields

Single Phase Fully Controlled Rectifier Control of DC Motor

$$i_a(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-(\omega t - \alpha)\cot\phi}\right] - \frac{E}{R_a}\left[1 - e^{-(\omega t - \alpha)\cot\phi}\right], \quad \text{for } \alpha \le \omega t \le \beta \qquad (7)$$

Since $i_a(\beta) = 0$, from Eq. (7)

$$\frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R_a} + \left[\frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{-(\beta - \alpha)\cot\phi} = 0 \qquad (8)$$

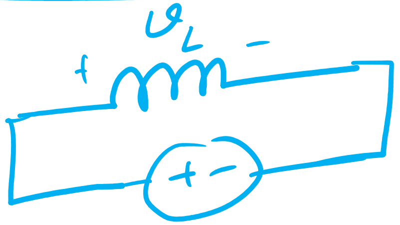
β can be the evaluated by iterative solution of Eq. (8).

Single Phase Fully Controlled Rectifier Control of DC Motor

Since the voltage drop across the armature inductance due to dc component of armature current is zero

$$V_a = E + I_a R_a \tag{9}$$

Fundamental voltage of motor.

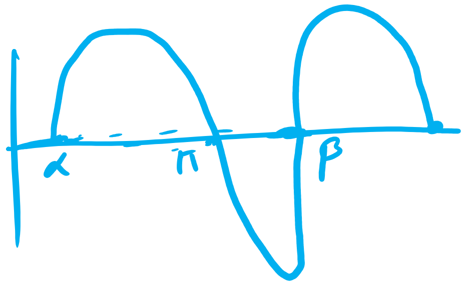


where V_a and I_a are respectively dc components of armature voltage and current, respectively.

α to β → First condition
β to π+α ⇒ 2nd condition.

$$V_a = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t \, d(\omega t) + \int_{\beta}^{\pi+\alpha} E \, d(\omega t)\right] \tag{10}$$

$$V_a = \frac{V_m(\cos\alpha - \cos\beta) + (\pi + \alpha - \beta)E}{\pi}$$



Single Phase Fully Controlled Rectifier Control of DC Motor

- Armature current consists of DC component I_a and harmonics.
- When flux is constant, only the DC component produces steady torque. Harmonics produce alternating torque components, the average value of which is zero.
- Therefore, the motor torque is still given by Eq. (7).
- From Eqs. (7), (8), (9) and (8)

$$\omega_m = \frac{V_m(\cos\alpha - \cos\beta)}{K(\beta - \alpha)} - \frac{\pi R_a}{K^2(\beta - \alpha)} T \qquad (11)$$

Single Phase Fully Controlled Rectifier Control of DC Motor

- Boundary between continuous and discontinuous conduction is reached when $\beta = \pi + \alpha$.

- Substituting $\beta = \pi + \alpha$ in Eq. (8) gives the critical value of speed ω_{mc} which separates continuous conduction from discontinuous conduction for a given α as

$$\omega_{mc} = \frac{R_a V_m}{ZK} \sin(\alpha - \phi) \left[\frac{1 + e^{-\pi \cot\phi}}{e^{-\pi \cot\phi} - 1} \right] \qquad (12)$$

ω_{mc} = critical speed of motor

Continuous Conduction Mode

Single Phase Fully Controlled Rectifier Control of DC Motor

From Fig. 3.

$$V_a = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha \qquad (13)$$

$v_a = \frac{2V_m}{\pi}\cos\alpha$

From Eqs. (7), (8), (9)

$$\omega_m = \frac{2V_m}{\pi K}\cos\alpha - \frac{R_a}{K^2}T \qquad (14)$$

$V_a = R_a + L_a\frac{di_a}{dt} + E$

$\omega \propto V_a$

$T \propto I_a$

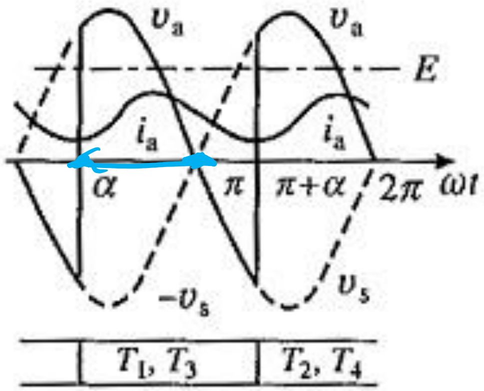


Fig. 3. Continuous conduction waveform.

Single Phase Fully Controlled Rectifier Control of DC Motor

- Speed torque curves for the drive are shown in Fig. 4.
- The ideal no load operation is obtained when $I_a = 0$.
- When both thyristor pairs (T_1, T_3) and (T_2, T_4) fail to fire, I_a will be zero.
- This will happen when $E > v_s$ throughout the period for which tiring pulses are present.
- Therefore, when $\alpha < \pi/2$, E should be greater or equal to V_m and when $\alpha > \pi/2$, E should be greater or equal to $V_m \sin \omega t$. Therefore, no load speeds are given by

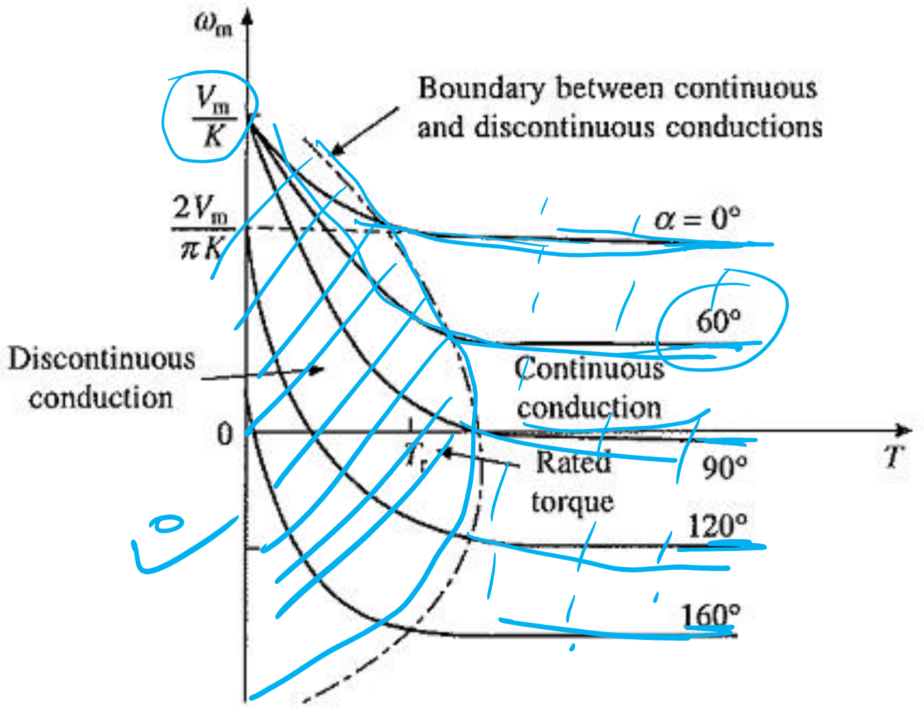


Fig. 4. Speed torque characteristics of single phase fully controlled rectifier Fed DC separately excited DC motor.

Single Phase Fully Controlled Rectifier Control of DC Motor

$$\omega_{m0} = \frac{V_m}{K}, \quad \text{for } 0 \le \alpha \le \pi/2 \qquad (15)$$

$$= \frac{V_m \sin\alpha}{K}, \quad \text{for } \pi/2 \le \alpha \le \pi \qquad (16)$$

- The maximum average terminal voltage ($2V_m/\pi$) is chosen equal to the rated motor voltage. ideal no-load speed of the motor when fed by a perfect direct voltage of rated value will then be ($2V_m/\pi K$).

- It is interesting is note that the maximum no load speed with rectifier control is ($\pi/2$) times this value.

Single Phase Fully Controlled Rectifier Control of DC Motor

- The boundary between continuous and discontinuous conduction is shown by dotted line (Fig. 4).
- For torques less than rated, a low power drive mainly operates in discontinuous conduction.
- In continuous conduction, the speed-torque characteristics are parallel straight lines, whose slope, according to (5.84), depends on the armature circuit resistance R_a.
- The effect of discontinuous conduction is to make speed regulation poor. This behavior can be explained from waveforms of Figs. 2 and 3.

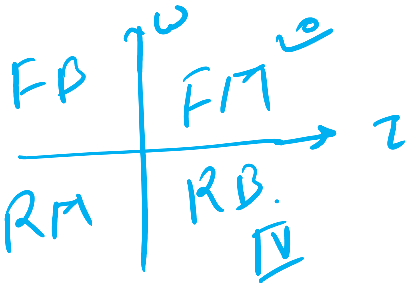
Single Phase Fully Controlled Rectifier Control of DC Motor

- In continuous conduction, for a given α, any increase in torque causes ω_m and E to drop so that I_a and T can increase.

- Average terminal voltage V_a remains constant. In discontinuous conduction, any increase in torque and accompanied increase in I_a causes β to increase and V_a to drop.

- Consequently, speed drops by a larger amount.

- The drive operates in quadrants I (forward motoring) and IV (reverse regenerative braking). These operations can be explained as follows:



Single Phase Fully Controlled Rectifier Control of DC Motor

- From Eq. (14), under the assumption of continuous conduction, dc output voltage of rectifier varies with α as shown in Fig. 5.

- When working in quadrant I, ω_m is positive and $\alpha \leq 90°$; and polarities of V_a and E are shown in Fig. 6.

- For positive I_a this causes rectifier to deliver power and the motor to consume it, thus giving forward motoring.

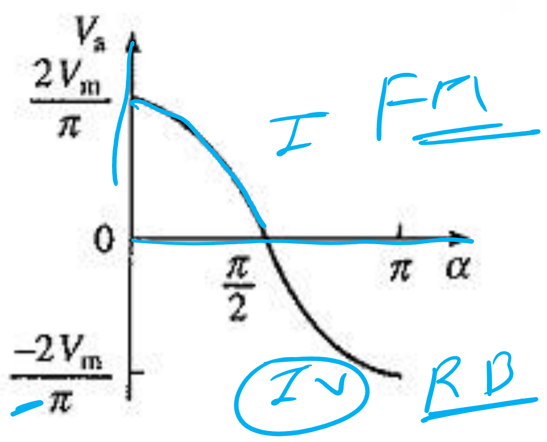


Fig. 5. V_a and α curve.

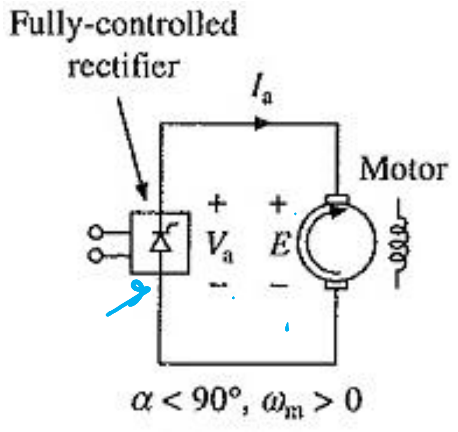


Fig. 6. Motoring mode.

Single Phase Fully Controlled Rectifier Control of DC Motor

- Polarities of E, I_a and V_a for quadrant IV operation are shown in Fig. 7.

- E has reversed due to reversal of ω_m. Since I_a is still in same direction, machine is working as a generator producing braking torque.

- Further due to $\alpha > 90°$, V_a is negative, suggesting that the rectifier now takes power from dc terminals and transfers it to ac mains.

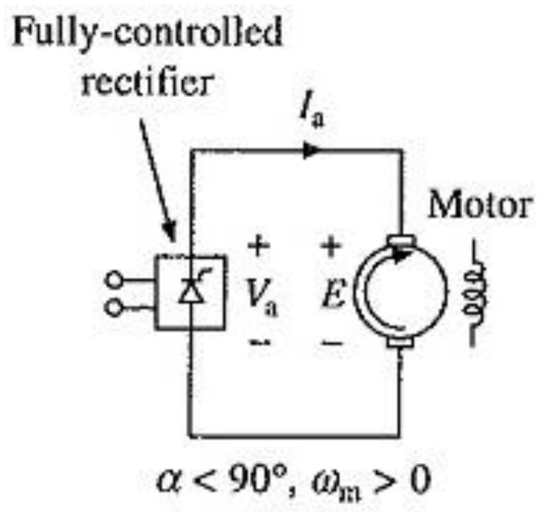


Fig. 7. Regenerative braking mode.

Single Phase Fully Controlled Rectifier Control of DC Motor

- This operation of rectifier is called **inversion** and the rectifier is said to operate as an inverter.

- Since generated power is supplied to the source in this operation, it is regenerative braking.

- Two-quadrant operation capability of the drive can be utilized only with overhauling loads or other active loads that can drive the motor in reverse direction.

- In a normal two-quadrant operation of a motor one needs forward motoring (quadrant I) and forward braking (quadrant II) which cannot be provided by the drive of Fig. 1.

Single Phase Half Controlled Rectifier Control of DC Separately Excited Motor

Single Phase Half Controlled Rectifier Control of DC Motor

- Single Phase Half Controlled Rectifier Control is shown in Fig. 1.

- In a cycle of source voltage defined by Eq. (1), T_1 receives a gate pulse from α to π and T_2 from $(\pi + \alpha)$ to 2π.

$$v_s = V_m \sin \omega t \quad (1)$$

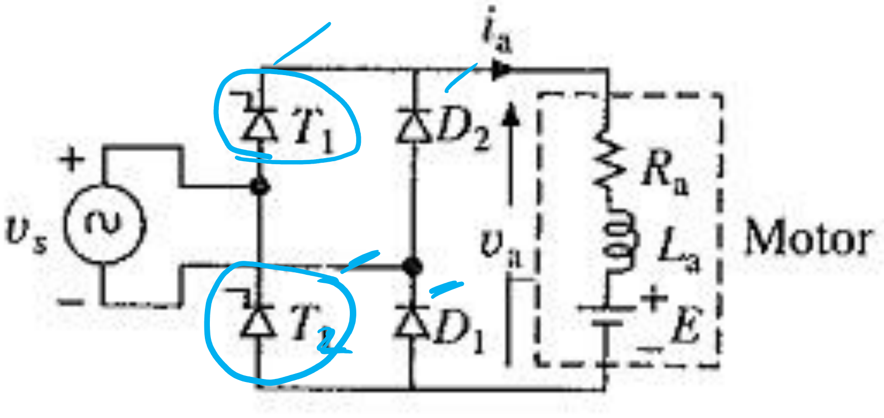


Fig. 1. Single phase half-controlled rectifier DC motor.

T1, T2, ⇒ Thyristors
D1, D2 ⇒ Diodes

At time operation
⇒ T1 and D1
⇒ T2 and D2

Single Phase Half Controlled Rectifier Control of DC Motor

- Motor terminal voltage and current waveforms for the dominant discontinuous and continuous conduction mode are shown in Figs. 2 and 3, respectively.

- In discontinuous conduction mode, when T_1 is fired at α, motor gets connected to the source through T_1 and D_1 and $v_a = v_s$.

- The armature current flows and D_2 gets forward biased at π. Consequently, armature current freewheels through the path formed by D_1 and D_2, and the motor terminal voltage is zero.

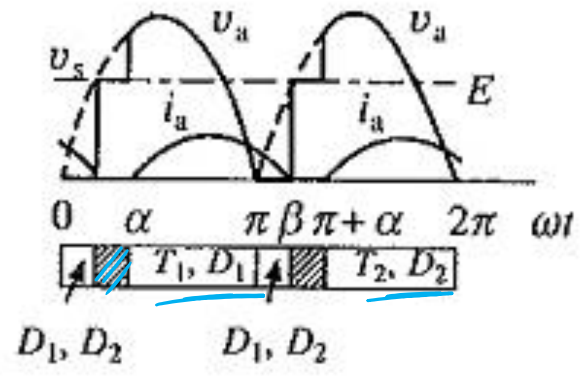


Fig. 2. Discontinuous conduction mode.

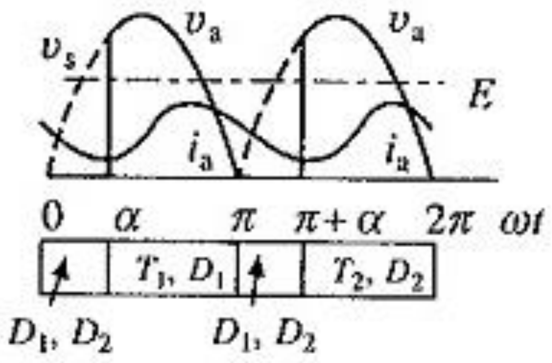


Fig. 3. Continuous conduction mode.

Single Phase Half Controlled Rectifier Control of DC Motor

- Conduction of D_2 reverse biases T_1 and turns it off. Armature current drops to 0 at β and stays zero until T_2 is fired at (π+ α).

- Similarly, the continuous conduction mode can be explained.

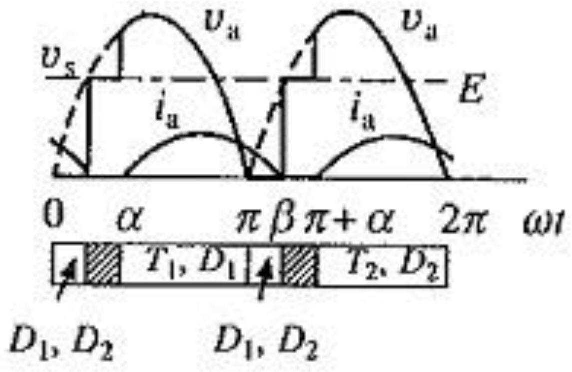


Fig. 2. Discontinuous conduction mode.

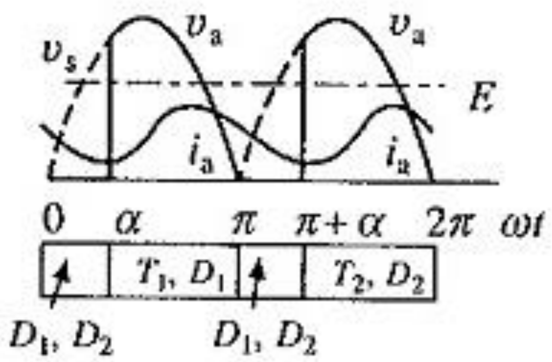


Fig. 3. Continuous conduction mode.

Discontinuous Conduction Mode

Single Phase Half Controlled Rectifier Control of DC Motor

- A cycle of motor terminal voltage consists of three intervals (Fig. 2):

1. Duty interval ($\alpha \le \omega t \le \pi$): Armature current is given by Eq. (2). Substitution of $\omega t = \pi$ in this equation gives $i_a(\pi)$.

$$i_a(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-(\omega t - \alpha)\cot\phi}\right] - \frac{E}{R_a}\left[1 - e^{-(\omega t - \alpha)\cot\phi}\right], \quad \text{for } \alpha \le \omega t \le \beta \qquad (2)$$

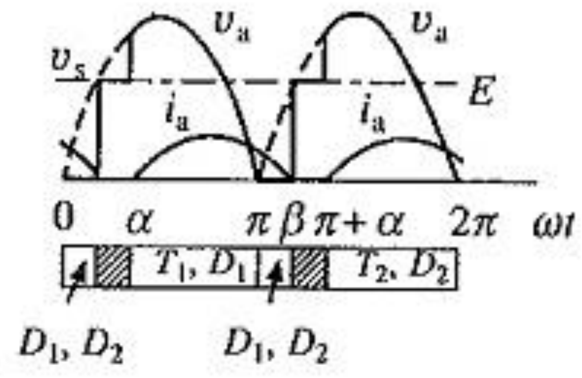


Fig. 2. Discontinuous conduction mode.

Single Phase Half Controlled Rectifier Control of DC Motor

2. Freewheeling interval ($\pi \leq \omega t \leq \beta$): Operation is governed by the following equation:

$$i_a R_a + L_a \frac{di_a}{dt} + E = 0 \qquad (3)$$

Solution of (3) subject to $i_a(\pi)$ as the initial current yields

$$i_a(\omega t) = \frac{V_m}{Z}\left[\sin\phi \cdot e^{-(\omega t - \pi)\cot\phi} - \sin(\alpha - \phi)\cdot e^{-(\omega t - \alpha)\cot\phi}\right] - \frac{E}{R_a}\left[1 - e^{-(\omega t - \alpha)\cot\phi}\right], \quad \text{for } \pi \leq \omega t \leq \beta \qquad (4)$$

Key Points from Today's Class

- Overview of Types of DC Motor Drives
- Speed Control of DC Motor Drives
- Armature Voltage Control of DC Motor using Transformer
- Controlled Rectifier Fed DC Drives

Key Points from Next Class

In the next class, we will be discussing on the

- ❖ Single Phase Fully Controlled Rectifier Control of DC Motor
- ❖ Single Phase Half Controlled Rectifier Control of DC Motor

Thank you so much for your attentions

Q & A