Tribhuvan University
Institute of Engineering
Pulchowk Campus
Accredited by University Grants Commission (UGC) Nepal 2020

Unit: I- Introduction to Electric Drives

Class-03:
1st January 2024

Presented by
Dr. Rajesh M. Pindoriya
rajeshpindoriya@ieee.org
Website: rmpindoriya.weebly.com

Subject Name
EE: Modelling and Control of Electric Drives

Discussed in the Previous Class

In the previous class discussed the following topics:

- ❖ Block Diagram of Electric Drives
- ❖ Classifications of Electric Drives
- ❖ Concepts and Elements of Electric Drives
- ❖ Torque-Speed Characteristics Curves

Lecture Outcomes

- ❖ Fundamental Torque Equation of Electric Drives
- ❖ Four-quadrant Operation of drive
- ❖ Equivalent Values of Drive Parameters
- ❖ Lecture remarks: Key points of today’s class

Choice of Electrical Drives

Choice of electrical drives means choosing the type of electrical drive that is suitable for the load or the system. It depends on the following factors:-

1. **Steady-state operation requirements**: Nature of speed-torque characteristics, speed regulation, speed range, efficiency, duty cycle, quadrants of operation, speed fluctuations if any, and ratings.
2. **Transient operation requirements**: Values of acceleration and deceleration, starting, braking, and reversing performance.
3. **Requirements are related to the source**: Type of source, and its capacity, magnitude of voltage, voltage fluctuations, power factor, harmonics and their effect on other loads, ability to accept regenerated power.
4. Capital and running cost, maintenance needs, life.
5. Space and weight restrictions if any.
6. Environment and location.
7. Reliability.

Fundamental Torque Equation of Electric Drives

- A motor generally drives a load through some transmission system. While the motor always rotates, the load may rotate or undergo a translational motion.
- Load speed may be different from that of a motor, and if the load has many parts, their speeds may be different and while some may rotate, others may go through a translational motion.

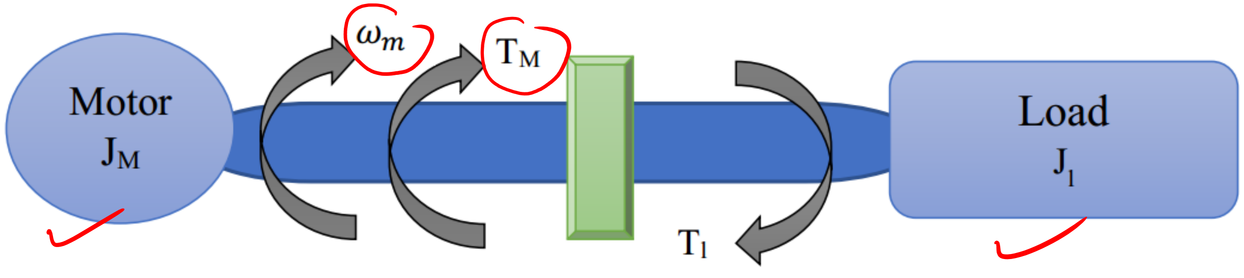


Fig. 1. Equivalent motor load system.

Notations	Meaning
J	The polar moment of inertia of the motor load system refers to the motor shaft in (kg-m^2)
ω_m	Instantaneous angular velocity of motor shaft in (rad/s)
T_M	The instantaneous value of developed motor torque in (N-m).
T_l	Instantaneous value of load torque referred to the motor shaft in (N-m)

Fundamental Torque Equation of Electric Drives

The fundamental torque equation is defined by Eq. (1)

$$T - T_l = \frac{d}{dt}(J\omega_m) = J\frac{d\omega_m}{dt} + \omega_m\frac{dJ}{dt} \quad (1)$$

Where;

J = polar moment of inertia of motor load system in kg-m^2
ω_m = instantaneous angular velocity of motor shaft in rad/s.
T = instantaneous value of developed motor torque in N-m.
T_l = instantaneous value of load torque in N-m.

τ_m, τ

Eq. (1) is applicable to variable inertia drives such as mine winders, reel drives, industrial robot.

For drives with constant inertia $\left(\frac{dJ}{dt}\right) = 0$. Therefore;

$\frac{dJ}{dt} = 0$

$$T = T_l + J\frac{d\omega_m}{dt} \quad (2)$$

Fundamental Torque Equation of Electric Drives

Eq. (1) shows that torque developed by motor is counter balanced by a load torque T_l and a dynamic torque $J\left(\frac{d\omega_m}{dt}\right)$. Torque component $J\left(\frac{d\omega_m}{dt}\right)$ is called the dynamic torque because it is present only during the transient operations. Drive accelerate or decelerates depending on weather T is greater of less than T_l.

A simple model of rotating electrical machines system is developed by following set of equations;

The total torque generated by machine is given by Eq. (3)

$$T = T_M - T_l \tag{3}$$

$T_m > T_e$

$T_m < T_e$

An equivalent moment of inertia is given by Eq. (4)

$$J_{eq} = J_M + J_l \tag{4}$$

Fundamental Torque Equation of Electric Drives

An acceleration given by Eq. (5)

$$a = \frac{T}{J_{eq}} \tag{5}$$

An angular speed is given by Eq. (6)

$$\omega_m = \int a \, dt \tag{6}$$

Rotor position is given by Eq. (7)

$$\theta = \int \omega_m dt \tag{7}$$

Where;

T_M is motor torque in N-m, a is acceleration of motor in m/s and θ is rotor position angle in rad.

Four Quadrant Operation of Electric Drive

- A motor operates in two modes:

1. **Motoring**
2. **Braking**

- ***In motoring,*** it converts electrical energy to mechanical energy, which supports its motion.

- ***In braking,*** it works as a generator converting mechanical energy to electrical energy, and thus, opposes the motion.

- Motor can provide motoring and braking operations for both forward and reverse directions.

Four Quadrant Operation of Electric Drive

- A four-quadrant or multiple-quadrant operation is required in industrial as well as commercial applications.

- These applications require both driving and braking, i.e., motoring and generating capability.

- Some of these applications include *electric traction systems, cranes and lifts, cable laying winders, and engine test-loading systems.*

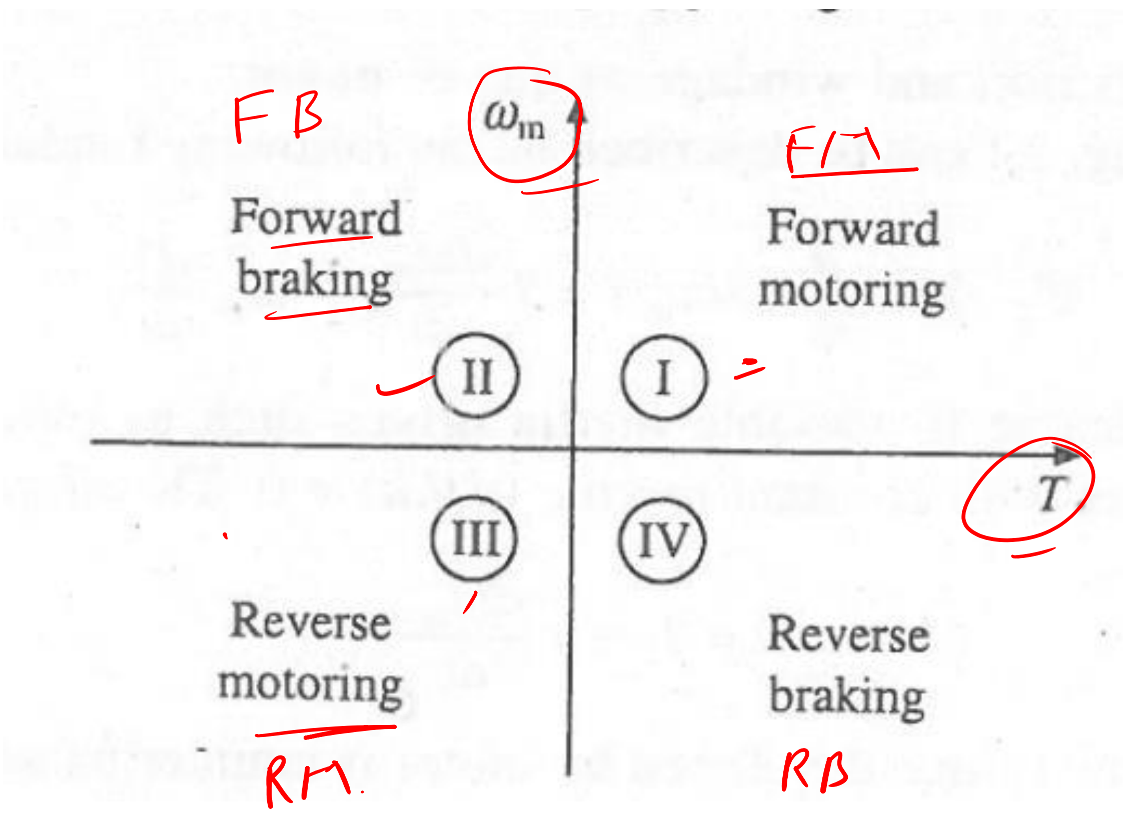


Fig. 1. Multi-quadrant operation of drives.

Four Quadrant Operation of Electric Drive

- Figure 1 shows the torque and speed coordinates for both forward (positive) and reverse (negative) motions of the Four Quadrant Operation of Motor Drive.

- Power developed by a motor is given by the product of speed and torque.

- In quadrant I, developed power is positive. Hence, the machine works as a motor supplying mechanical energy. Operation in quadrant I is, therefore, called **forward motoring.**

- In quadrant II, power is negative. Hence, the machine works under braking opposing the motion. Therefore, operation in quadrant II is known as **forward braking**.

- Similarly, operations in quadrants III and IV can be identified as reverse motoring and braking respectively.

Four Quadrant Operation of Electric Drive

- For consideration of the four-quadrant operation of motor drives, it is useful to *establish suitable conventions about the signs of torque and speed.*

- **Motor speed is considered positive when rotating in the forward direction.**

- For drives that operate only in one direction, forward speed will be their normal speed.

- In loads involving up-and-down motions, the speed of the motor which causes upward motion is considered forward motion.

- For reversible drives, forward speed is chosen arbitrarily.

- Then the rotation in the opposite direction gives reverse speed which is assigned the negative sign.

Four Quadrant Operation of Electric Drive

- Positive motor torque is defined as the torque that produces acceleration or the positive rate of change of speed in a forward direction.

- According to Eq. (1), positive load torque is opposite in direction to the Positive motor torque. Motor torque is considered negative if it produces deceleration.

$$T = T_l + J\frac{d\omega_m}{dt} \qquad (1)$$

Four Quadrant Operation of Electric Drive

- For a better understanding of the above notations, let us consider the operation of a hoist in Four Quadrant Operation of Motor Drive as shown in Fig. 2.

- Directions of motor and load torques, and direction of speed are marked by arrows.

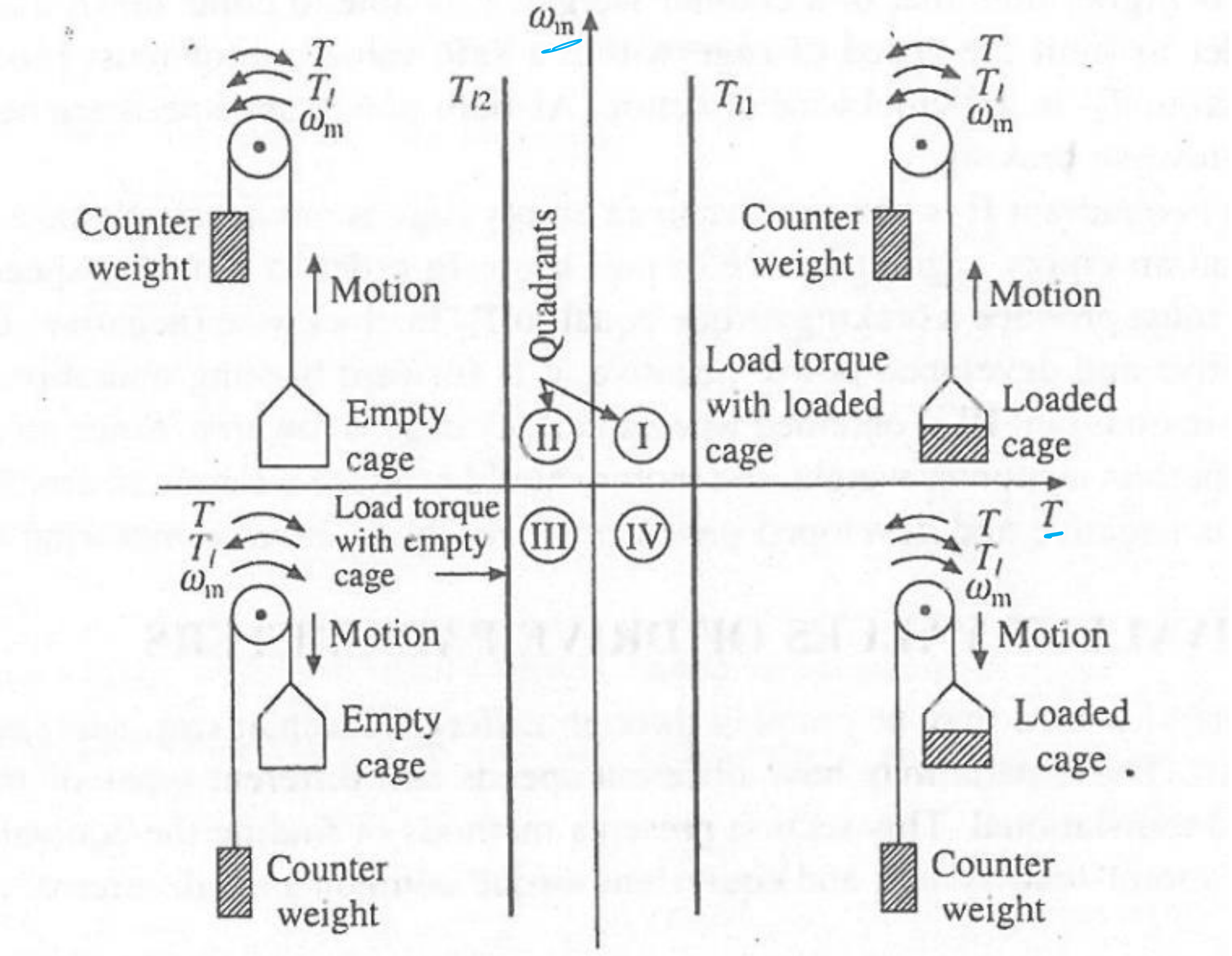
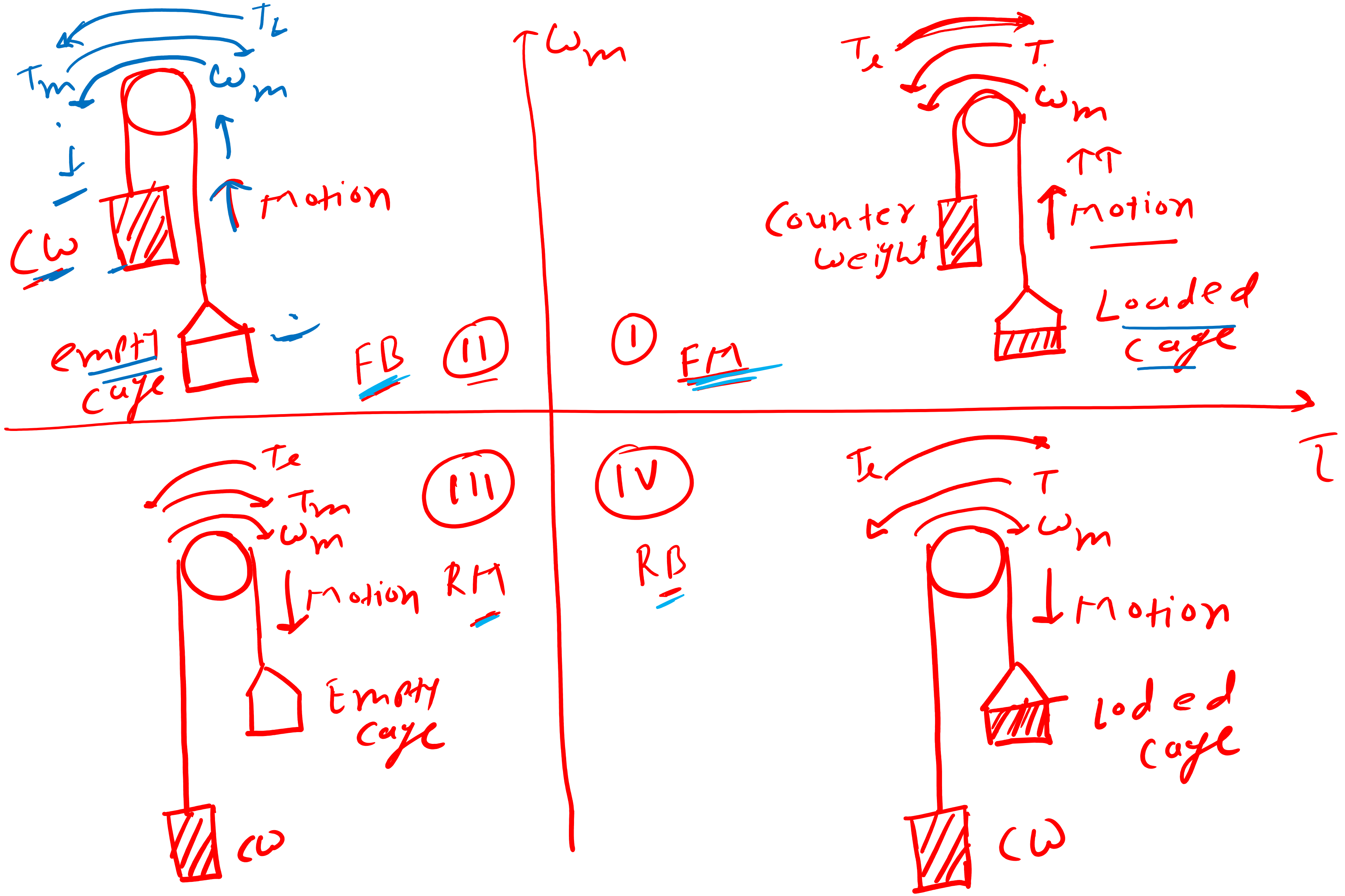


Fig. 2. Multi-quadrant operation of a motor driving a hoist load.

Four Quadrant Operation of Electric Drive



Four Quadrant Operation of Electric Drive

- A hoist consists of a rope wound on a drum coupled to the motor shaft.
- One end of the rope is tied to a cage which is used to transport man or material form one level to another level.
- Other end of the rope has a counter weight.
- Weight of the counter weight is chosen to be higher than the weight of an empty cage but lower than of a fully loaded cage.

Four Quadrant Operation of Electric Drive

- The forward direction of motor speed will be one which gives upward motion of the cage.

- Speed-torque characteristics of the hoist load are also shown in Fig. 2.

- Though the positive load torque is opposite in sign to the positive motor torque, according to Eq. (1), it is convenient to plot it on the same axes.

- The load-torque curve drawn in this manner is, in fact, negative of the actual.

Four Quadrant Operation of Electric Drive

- Load torque has been shown to be constant and independent of speed. This is nearly true with a low speed hoist where forces due to friction and windage can be considered to be negligible compared to those due to gravity.

- Gravitational torque does not change its sign even when the direction of driving motor is reversed. Load torque line T_{l1} in quadrants I and IV represents speed-torque characteristic for the loaded hoist.

- This torque is the difference of torques due to loaded hoist and counter weight. The load torque line T_{l2} in quadrants II and III is the speed-torque characteristic for an empty hoist.

- This torque is the difference of torques due to counter weight and the empty hoist. Its sign is negative because the weight of a counter weight is always higher than that of an empty cage.

Four Quadrant Operation of Electric Drive

- ***The quadrant I*** operation of a hoist requires the movement of the cage upward, which corresponds to the positive motor speed which is in anticlockwise direction here.

- This motion will be obtained if the motor produces positive torque in anticlockwise direction equal to the magnitude of load torque T_{l1}.

- Since developed motor power is positive, this is forward motoring operation.

$P_m = \tau \times \omega$

Four Quadrant Operation of Electric Drive

- ***Quadrant IV*** operation is obtained when a loaded cage is lowered.
- Since the weight of a loaded cage is higher than that of a counter weight, it is able to come down due to the gravity itself.
- In order to limit the speed of cage within a safe value, motor must produce a positive torque T equal to T_{l2} in anticlockwise direction.
- As both power and speed are negative, drive is operating in reverse braking.

Four Quadrant Operation of Electric Drive

- Operation in ***quadrant II*** is obtained when an empty cage is moved up.

- Since a counter weight is heavier than an empty cage, it is able to pull it up. In order to limit the speed within a safe value, motor must produce a braking torque equal to T_{l2} in clockwise (negative) direction.

- Since speed is positive and developed power negative, it is forward braking operation.

Four Quadrant Operation of Electric Drive

- Operation in ***quadrant III*** is obtained when an empty cage is lowered.

- Since an empty cage has a lesser weight than a counter weight, the motor should produce a torque in clockwise direction.

- Since speed is negative and developed power positive, this is reverse motoring operation.

Equivalent Values of Drive Parameters

Equivalent Values of Drive Parameters

- Different parts of a load may be coupled through different mechanisms, such as gears, V-belts and crankshaft.

- These parts may have different speeds and different types of Motions such as rotational and translational. This section presents methods of motor design Parameters finding the equivalent moment of inertia (J) of the motor-load system and equivalent torque components, all referred to as motor shaft.

1. **Loads with Rotational Motion**
2. **Loads with Translational Motion**
3. **Measurement of Moment of Inertia**

Equivalent Values of Drive Parameters

1. Loads with Rotational Motion:

- Let us consider a motor driving two loads, one coupled directly to its shaft and other through a gear with n and n_1 teeth as shown in Fig. 3.

- Let the moment of inertia of motor and load directly coupled to its shaft be J_0, motor speed and torque of the directly coupled load be ω_m and T_{l0} respectively.

- Let the moment of inertia, speed and torque of the load coupled through a gear be J_1, ω_{m1} and T_{l1} respectively.

$J_0 = J_m + J_{l1}$

$J_1 = J_{l2}$

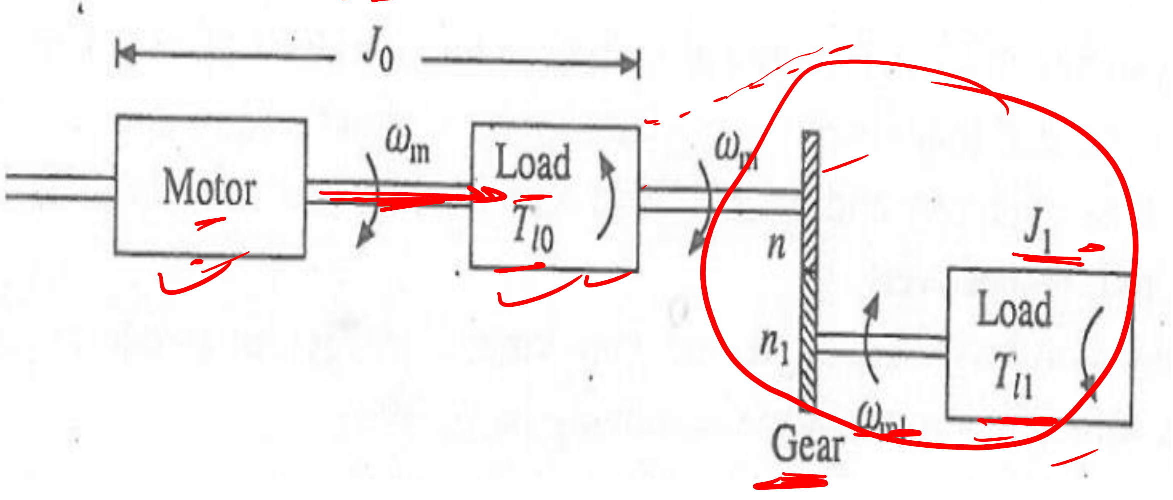


Fig. 3. Load with rotational motion.

Load- 01

Load - 02

Equivalent Values of Drive Parameters

Now,

$$\boxed{\frac{\omega_{m1}}{\omega_m} = \frac{n}{n_1} = a_1} \qquad (1)$$

where a_1 is the gear tooth ratio.

➢ If the losses in transmission are neglected, then the kinetic energy due to equivalent inertia must be the same as the kinetic energy of various moving parts. Thus

$$\frac{1}{2}J\omega_m^2 = \frac{1}{2}J_0\omega_m^2 + \frac{1}{2}J_1\omega_{m1}^2 \qquad (2)$$

Input = output

kinetic energy = $\frac{1}{2}J\omega_m^2$

$J = J_0 + J_1$

Equivalent Values of Drive Parameters

From equations (1) and (2)
Divided by ω^2

$$J = J_0 + a_1^2 J_1 \qquad (3)$$

Power at the loads and motor must be the same.

$P_m = \tau \times \omega$

If the transmission efficiency of the gears be η_1, then

η_1

$$T_l \omega_m = T_{l0} \omega_m + \frac{T_{l1} \omega_{m1}}{\eta_1} \qquad (4)$$

where T_l is the total equivalent torque referred to motor shaft.
From Eqs. (3) and (4)

Equivalent Values of Drive Parameters

$$T_l = T_{l0} + \frac{a_1 T_{l1}}{\eta_1} \quad (5)$$

Divide by ω_m

If in addition to load directly coupled to the motor with inertia J_0 there are m other loads with moment of inertias J_1, J_2, . . . , J_m and gear teeth ratios of a_1, a_2, . . . a_m then

$$J = J_0 + a_1^2 J_1 + a_2^2 J_2 + \ldots + a_m^2 J_m \quad (6)$$

Equivalent Values of Drive Parameters

If m loads with torques T_{l1}, T_{l2}, . . . , T_{lm} are coupled through gears with teeth ratios a_1, a_2, . . . a_m and transmission efficiencies η_1, η_2 , . . . , η_m, in addition to one directly coupled, then

$$T_l = T_{l0} + \frac{a_1 T_{l1}}{\eta_1} + \frac{a_2 T_{l2}}{\eta_2} + \ldots + \frac{a_m T_{lm}}{\eta_m} \qquad (7)$$

If loads are driven through a belt drive instead of gears, then, neglecting slippage, the equivalent inertia and torque can be obtained from Eqs. (6) and (7) by considering a_1, a_2, . . . , a_m each to be the ratios of diameters of wheels driven by motor to the diameters of wheels mounted on the load shaft.

Equivalent Values of Drive Parameters

2. Loads with Translational Motion

$J_0 = J_m + J_{l0}$

- Let us consider a motor driving two loads, one coupled directly to its shaft and other through a transmission system converting rotational motion to linear motion (Fig. 4).

- Let moment of inertia of the Motor Design Parameters and load directly coupled to it be J_0, load torque directly coupled to motor be T_{l0}, and the mass, velocity and force of load with translational motion be M_1 (kg), υ_1 (m/sec) and F_1 (Newtons), respectively.

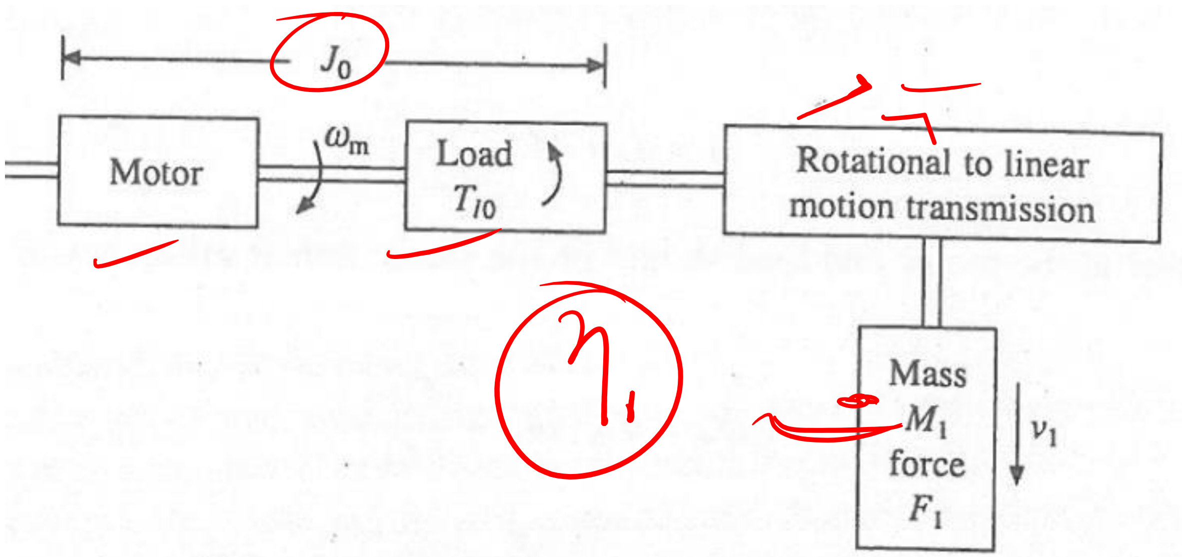


Fig. 4. Load with translational and rotational motion.

Equivalent Values of Drive Parameters

- If the transmission losses are neglected, then kinetic energy due to equivalent inertia J must be the same as kinetic energy of various moving parts. Thus

$\frac{1}{2} J \omega_m^2$

$$\frac{1}{2} J\omega_m^2 = \frac{1}{2} J_0\omega_m^2 + \frac{1}{2} M_1 v_1^2$$

$$J = J_0 + M_1\left(\frac{v_1}{\omega_m}\right)^2 \qquad (8)$$

Similarly, power at the motor and load should be the same, thus if efficiency of transmission be η_1

$P = \tau \times \omega$

$$T_l\omega_m = T_{l0}\cdot\omega_m + \frac{F_1 v_1}{\eta_1}$$

$$\boxed{T_l = T_{l0} + \frac{F_1}{\eta_1}\left(\frac{v_1}{\omega_m}\right)} \qquad (9)$$

Equivalent Values of Drive Parameters

If, in addition to one load directly coupled to the motor shaft, there are m other loads with translational motion with velocities $\upsilon_1, \upsilon_2, \ldots \upsilon_m$ and masses $M_1, M_2, \ldots, M_m$, respectively, then

$$J = J_0 + M_1\left(\frac{\upsilon_1}{\omega_m}\right)^2 + M_2\left(\frac{\upsilon_2}{\omega_m}\right)^2 + \ldots + M_m\left(\frac{\upsilon_m}{\omega_m}\right)^2 \qquad (10)$$

$$T_l = T_{l0} + \frac{F_1}{\eta_1}\left(\frac{\upsilon_1}{\omega_m}\right) + \frac{F_2}{\eta_2}\left(\frac{\upsilon_2}{\omega_m}\right) + \ldots + \frac{F_m}{\eta_m}\left(\frac{\upsilon_m}{\omega_m}\right) \qquad (11)$$

Equivalent Values of Drive Parameters

3. Measurement of Moment of Inertia:

- Moment of inertia can be calculated if dimensions and weights of various parts of the load and Motor Design Parameters are known. It can also be measured experimentally by retardation test.

- In retardation test, the drive is run at a speed slightly higher than rated speed and then the supply to it is cut off. Drive continues to run due to kinetic energy stored in it and decelerates due to rotational mechanical losses. Variation of speed with time is recorded.

speed	time
1500	1 sec
1400	2 sec
1000	4 sec

Equivalent Values of Drive Parameters

At any speed ω_m power P consumed in supplying rotational losses is given by

$$P = \text{Rate of change of kinetic energy}$$

$$= \frac{d}{dt}\left(\frac{1}{2} J\omega_m^2\right) = J\omega_m \frac{d\omega_m}{dt} \qquad (12)$$

- From retardation test $d\omega_m/dt$ at rated speed is obtained. Now drive is reconnected to the supply and run at rated speed and rotational mechanical power input to the drive is measured.

- This is approximately equal to P.

Equivalent Values of Drive Parameters

- Now J can be calculated from Eq. (12).
- Main problem in this method is that rotational mechanical losses cannot be measured accurately because core losses and rotational mechanical losses cannot be separated.
- In view of this, retardation test on a dc separately excited motor or a synchronous motor is carried out with field on.
- Now core loss is included in the rotational loss, which is now obtained as a difference of armature power input and armature copper loss.
- In case of a wound rotor induction motor, retardation test can be carried out by keeping the stator supply and opening the rotor winding connection.

Equivalent Values of Drive Parameters

- J can be determined more accurately by obtaining speed time curve from the retardation test as above and also rotational losses vs speed plot as shown in Fig. 6.

- Using these two plots, rotational losses vs time plot can be obtained, e.g. for time t_1, ω_{m1} is found from the retardation plot. the

P = time, speed

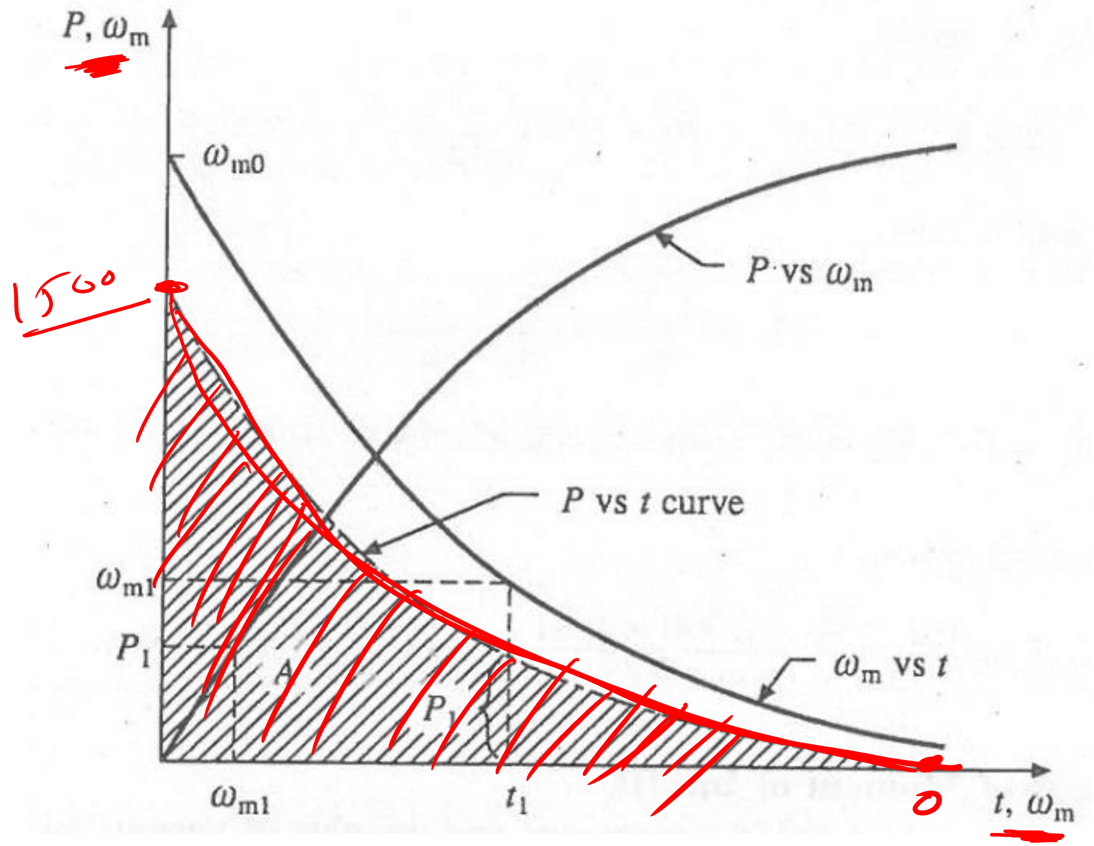


Fig. 6. Graphical method of determination of equivalent moment of inertia

Equivalent Values of Drive Parameters

- Then for this speed rotational loss P_1 is obtained from the plot of rotational loss vs speed and plotted against t_1.

- Area A enclosed between the rotational loss vs t plot and the time axis (shaded area), is the kinetic energy dissipated during retardation test.

- If initial speed of the drive during retardation test was ω_{m0} then

$$\frac{1}{2} J \omega_{m0}^2 = A \qquad (13)$$

$$\frac{1}{2} J \omega_m^2 = A$$

Key Points from Today's Class

- ❖ Fundamental Torque Equation of Electric Drives
- ❖ Four-quadrant Operation of drive
- ❖ Equivalent Values of Drive Parameters

Key Points from Next Class

In the next class, we will be discussing on the

- Dynamics of Motor-Load Combination
- Steady State Stability of an Electric Drive System
- Load Equalization

Thank you so much for your attentions

Q & A